



**FIRST TERM FIRST YEAR (1<sup>ST</sup> TERM) B.E.(ME, EL, ES, TL, BM, CS, SW, CH, PG, MN,  
MT & IN) FOR 07-BATCH REGULAR EXAMINATION 2007.**

**APPLIED CALCULUS**

Dated: 22-05-2007.

Time Allowed: 03 Hours.

Max.Marks-80.

**NOTE. ATTEMPT ANY FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.**

**Q.No.**

01. (a) Define function. Does the area of circle,  $A=\pi r^2$  defines the concept of function? The cost of producing  $x$  chips of computer is given by  $C(x)=x^2+5x+5$  rupees, find the cost of producing (i) 5 chips (ii) 5<sup>th</sup> chip .
- (b) Without using L-hospital rule Evaluate any two of the following:
- (i)  $\lim_{x \rightarrow 0} \frac{4^x - 2^x}{x}$       (ii)  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sin^{-1}x}$       (iii)  $\lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h}$
- (c) Find the value of  $U$  so that given function  $f(x)$  is continuous. Where
- $$f(x) = \begin{cases} x^2 + a & \text{if } -2 < x < 2 \\ 6 & \text{at } x = 2 \\ x - 2 & \text{if } x > 2 \end{cases}$$
02. (a) Define derivative of function and its physical meaning with suitable example. If two resistors are connected in series and if  $R_1$  is increasing at rate of  $5\Omega/\text{sec}$  and  $R_2$  is decreasing at rate of  $2\Omega/\text{sec}$ , find change in  $R$ .
- (b) Differentiate  $(\tan x)^{\cot x}$  w.r.t  $(\cot x)^{\tan x}$ .
- (c) A dynamite blast blows a heavy rock with a launch velocity of 160 ft/sec, it reaches at height of  $s=100t-5t^2$  ft after  $t$  sec, find  
(i) how high does the rock go? (ii) when it reaches the ground again?
03. (a) State Rolles Theorem and give its geometrical interpretation. Find  $c$  by Rolles Theorem (if possible) for  $f(x) = x^2-7x+12$  on  $[3,4]$ .
- (b) Define tangent and normal to the curve  $y=f(x)$ . Find the equation of tangent and normal for  $x^2y+y^2x=10$  at the point  $(1,2)$ .
- (c) Assume that the spider is spinning a circular web and suppose that the radius of the web is changing at rate of 8mm/hour, find the rate at which the area and the circumference of web is increasing when radius is 25mm.
04. (a) State and Prove Leibnitz Theorem, hence find the  $n$ th derivative of  $x^2y_2$ .
- (b) Define curvature and radius of curvature. What will be the curvature of any straight line? Find the radius of curvature for  $x^2y-x^2-y^2=0$  at  $(2,2)$ .
- (c) Define Maxima and Minima of  $f(x)$ . Discuss working rules for maxima and minima for  $f(x,y)=0$ . The output  $P$  of battery is given by  $P=VI-RI^2$ , where  $V$  is voltage in volts,  $I$  is current in amperes and  $R$  is resistance in ohms. Find the maximum current and maximum output of battery when  $V=4$  volts and  $R=8$  ohms.

Cont'd on P/-2....

05. (a) Define homogenous function with suitable examples. State and prove Eulers Theorem. Verify Eulers Theorem for  $u = \sqrt{x^2 + y^2}$ .
- (b) If  $u = \ln \left( \frac{x^6 + y^6}{x^2 + y^2} \right)$ . prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 4$ .
- (c) Define total differential for  $f(x,y)=0$ . According to heat law, a rectangular plate expands in such a way that its length changes from 5.4 cm to 5.7 cm and its breadth changes from 4cm to 4.3 cm, find approximate change in the area of the plate.
06. (a) Evaluate any two of the following:  
(i)  $\int \frac{\cos x dx}{(1 + \sin x)(2 + \sin x)}$  (ii)  $\int \frac{dx}{(x+2)\sqrt{(x+3)}}$  (iii)  $\int e^x \left( \frac{1 - \sin x}{1 - \cos x} \right) dx$
- (b) Prove that  $\int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x} = \frac{\pi^2}{2ab}$
- (c) Define improper integrals, hence evaluate  $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ , also evaluate  $\beta(3,4)$ .
07. (a) Define integrand, integration and integral. To test learning a teacher asks the students to memorize table of 1 to 20. Assume that the rate at which table is memorized is given by  $\frac{dm}{dt} = 5.4e^{-0.3t}$  tables/minute, where m is number of tables memorized and t is time in minutes, how many tables will be memorized after 7 minutes.
- (b) Find the volume of the solid of revolution by revolving the curve  $y=2x^2$  about the x-axis on the interval [1,2].
- (c) The project rate of increase in Mehran University of Engineering and Technology is estimated by  $\frac{dE}{dt} = 3000(t+4)^{-2/3}$ ,  $t > 0$ , where E(t) is projected enrolment in t years. If the enrollment is 300 at  $t=0$ , find projected enrollment of 5 years.
08. (a) What do you mean by scalar and vector point function? Define gradient, divergence and curl. Find the unit normal to the surface  $x^2+y^2+z^2=25$  at (2,2,1).
- (b) Define Solenoidal and irrotational vectors. Find the constant P so that the vector  $(3x+y)\vec{i} + (py+3z)\vec{j} + (5z+3x)\vec{k}$  is solenoid. Also show that the vector  $\vec{u} = x\vec{i} + y\vec{j} + z\vec{k}$  is irrotational.
- (c) A particle moves in a straight line so that its components are given by  $x=e^t$ ,  $y=\sin t$  and  $z=\cos t$  Find the velocity and acceleration of particle at  $t=0$ , also find the magnitude of velocity and acceleration at  $t=0$  **OR**  
Prove that  $\text{div } \vec{r} = 3$  and  $\nabla \cdot \vec{r}^n = n r^{n-2} \vec{r}$ , where  $\vec{r}$  is position vector in  $R^3$ .



**FIRST TERM FIRST YEAR (1<sup>ST</sup> TERM) B.E.(ME, EL, ES, TL, CS, SW, CH, PG, MN, MT & IN) REGULAR EXAMINATION 2009 OF 09-BATCH.**

**APPLIED CALCULUS**

Dated: 19-05-2009.

Time Allowed: 03 Hours.

Max.Marks-80.

NOTE. ATTEMPT ANY FIVE QUESTIONS.

<u>Q.No.</u>	<u>Marks</u>
01 (a) Define continuous and discontinuous function.	(05)
A psychologist needs volunteers for collecting the data for his experimental work. He offers to pay Rs.300/= per hour for volunteer who works up to 8 hours daily. Those who work more than 8 hours are paid Rs.400/= for additional hour. Write down the function $W(x)$ , where $x$ represents the hours worked. Find $W(10)$ and $W(20)$ .	
(b) Define continuity of a function $f(x)$ . The amount of drug that remains in a person's bloodstream $t$ hours after being injected is given by:	(05)
$A(t) = \begin{cases} t^2 + 3, & 0 < t \leq 3 \\ 4t, & 3 < t \leq 6 \\ t, & 6 < t < 9 \end{cases}$	
Is the amount of drug in person's bloodstream continuous at time $t = 3$ hours and $t = 6$ hours after it was being taken by patient?	
(c) Evaluate any TWO of the following limits:	(06)
(i) $\lim_{x \rightarrow 2+0} \sqrt{\frac{4+x^2}{6-5x-x^2}}$ (ii) $\lim_{x \rightarrow \pi} (\sin x)^{\tan x}$ (iii) $\lim_{x \rightarrow 1} \left( \frac{1}{x-1} - \frac{3}{x^3-1} \right)$	
02 (a) A bicycle manufacturer estimates that it can price its bicycles at $P=140-0.02x$ dollars each, where $x$ is the number of bicycles sold. The cost of producing bicycles is $900-0.01x^2$ dollars. Determine the marginal profit when 20 bicycles are made. What is the marginal profit on the sale of 20 <sup>th</sup> bicycle?	(05)
(b) A rocket is launched straight up. There is an observation station 7 miles from the launch site. At what rate is the distance between the rocket and the station increasing when the rocket is 24 miles high and traveling at 200 miles per hour?	(06)
(c) If $y \sin^{-1}x - x \tan^{-1}y = 0$ , find $dy/dx$ .	(05)
03 (a) State Leibniz Theorem. Use it to show that if $y = \sin(2 \arccos x)$ , then $(1-x^2) y_{n+2} = (2n+1)x y_{n+1} + (n^2-4)y_n$ .	(05)
(b) Find the $n$ th derivative of $\frac{x}{(x-1)(x-2)}$	(05)
(c) Show that the function $4 = \tan^{-1} \frac{2xy}{x^2-y^2}$ is harmonic function.	(06)

- 04 (a) Define asymptote of a function  $y = f(x)$ . Find all possible asymptotes of  $2xy + 2y = (x - 2)^2$ . (06)
- (b) Find the two points where the curve  $x^2 + xy + y^2 = 7$  crosses the  $x$  - axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents? (05)
- (c) Find the radius of curvature of the curve  $9x^2 + 16y^2 = 180$  at  $(2, 3)$ . (05)
- 05 (a) A wire 50 centimeters long is cut into two pieces. One piece (call its length  $x$ ) will be bent to form a square. The other piece (of length  $50 - x$ ) will be bent to form a circle. How much wire should be used for the square if the total area (square plus circle) is to be the smallest possible? (06)
- (b) **State the Euler theorem. If  $u = \sec^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$ , show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \cot u$ .** (05)
- (c) **The dimensions of a rectangular block of wood were found to be 10, 12, and 20 inches, with a possible error of 0.05 in each of the measurements. Find (approximately) the error in the surface area of the block caused by errors in the individual measurements.** (05)
- 06 (a) **At a certain instant the radius of a right circular cylinder is 6 inches and is increasing at the rate 0.2 in/sec while the altitude is 8 inches and is decreasing at the rate 0.4 in/sec. Find the time rate of change of the volume at that instant.** (05)
- (b) A manufacturer of aquarium wants to make a large rectangular box - shaped aquarium that will hold  $64 \text{ ft}^3$  of water. If the material for the base costs \$20 per square foot and the material for the sides costs \$10 per square foot, find the dimensions for which the cost of the materials will be the least. (05)
- (c) Evaluate any TWO of the following integrals. (06)
- (i)  $\int \ln \frac{(1+x^2)}{x^2} dx$  (ii)  $\int \frac{(1+x^2)}{(x-3)(x^2+4)} dx$  (iii)  $\int \frac{dx}{(x+2)\sqrt{x+3}}$
- 07 (a) The enrolment  $E$  of students in a university increases at the rate of  $dE/dt = 3000(t+4)^{-2/3}$ , where  $t$  is the time in years. If the enrolment at time  $t = 0$  is 1500, find the enrolment of students after 15 years. (05)
- (b) Find the area bounded by the curves:  $y = x^3$  and  $y = x$ . Show the area graphically. (05)
- (c) Show that:  $\Gamma(1/2) = \sqrt{\pi}$  and hence evaluate  $\beta\left(\frac{7}{2}, \frac{5}{2}\right)$ . (06)

**08 (a) An acceleration of a particle is given by:  $\vec{f}(t) = t^3\hat{i} + t^2\hat{j} - (t-2)\hat{k}$ . (05)**

**Find the velocity and displacement vector given that  $\vec{f}'(0) = \hat{i} + \hat{j} + \hat{k}$   
and  $\vec{f}(0) = 2\hat{i} + 3\hat{j} - \hat{k}$ .**

(b) Define gradient of a scalar function. Also give its geometrical interpretation. (06)

Find a unit vector normal to the surface  $\varphi(x, y, z) = x^2y^2 + xy^2z + xyz$  at  $(1, 2, 1)$ .

(c) Evaluate  $\nabla^2 \ln r$  where  $\vec{r}$  is a position vector. (05)

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**FIRST TERM FIRST YEAR (1<sup>ST</sup> TERM) B.E.(ME, EL, ES, TL, BM, CS, SW, CH, PG, MN,  
MT & IN) REGULAR EXAMINATION 2008 OF 08-BATCH.**

**APPLIED**

Dated: 20-05-2008.

Time Allowed: 03 Hours.

Max.Marks-80.

NOTE. ATTEMPT ANY FIVE QUESTIONS.

<u>Q.No.</u>		<u>Marks</u>
1 (a)	The cost on producing $x$ radios is $C(x) = 0.4x^2 + 7x + 95$ \$. The revenue received is $R(x) = 40x$ \$. What is the profit function? Find $P(24)$ and $P(25)$ . What is the profit on the sale of 25 <sup>th</sup> radio?	(04)
(b)	The monthly charge (in dollars) for $x$ kilowatt hours (KWK) of electricity used by a commercial customer is given by the following function:	(04)
	$C(x) = \begin{cases} 5.72 + 0.7109x, & 0 < x \leq 5 \\ 22.19 + 0.7109x, & 5 < x \leq 750 \\ 20.795 + 0.1058x, & 750 < x < 1500 \\ 131.345 + 0.0321x & x > 1500 \end{cases}$	
	Is the monthly charge continuous at the consumption of 5 KWH and 1500 KWH of the electricity?	
(c)	Evaluate any TWO of the following:	(08)
	(i) $\lim_{x \rightarrow \pi} (\cot x)^{\sin x}$ , (ii) $\lim_{x \rightarrow 0} \frac{4^x - 2^x}{x}$ , (iii) $\lim_{x \rightarrow 0} (e^x - 1) \cot x$	
2 (a)	Soya Supreme ghee company produces bags of 5 kilo ghee but due to daily increase in prices the company decides to stop the production when marginal cost of ghee reaches Rs. 700 .If the cost function is given by $C(x) = 0.01x^2 + 200x + 50$ rupees how many bags of ghee the company produces before it halts the production?	(05)
(b)	A labor is using 20 m long ladder for painting a bungalow. If the bottom of ladder slips at rate of 2/3 m/sec, at what rate the top of ladder sliding down the wall if top of ladder is 12 m from the ground.	(05)
(c)	State the Leibnitz's theorem and use it to show that if $y = e^{-3\sin^{-1}x}$ , then	(06)
	$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + 9)y_n = D$	
3 (a)	Show that the function $f(x, y) = \tan^{-1}\left(\frac{2xy}{x^2 - y^2}\right)$ satisfies the Laplace equation	(05)
	$f_{xx} + f_{yy} = 0$ .	
(b)	State the Mean Value Theorem and give its geometrical interpretation.	(06)
	Verify the Mean Value Theorem for the function $f(x) = x(x-1)(x-2)$ on $\left[0, \frac{1}{2}\right]$	
(c)	Expand $\ln(x+1)$ into Taylor series about $x = 1$ . Also expand $e^x$ into Maclurin's series.	(05)

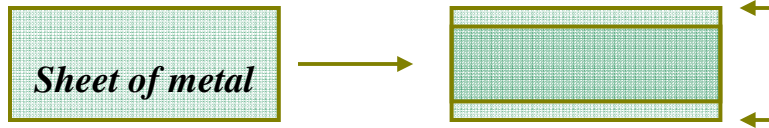
- 4 (a) Define tangent and normal to any curve .Find the equation of tangent for  $x(x^2 + y^2) - y^2 = 0$  at  $x = 1/2$ . (06)

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- 4 (b) Find the radius of curvature at the given point to the curve: (06)  
 $x^3 + y^3 = 3axy$ ;  $(3a/2, 3a/2)$

- (c) A builder plans to construct a gutter from a long sheet of metal by making two folds of equal size (see the figure). The folds are made to create perpendicular sides. (04)



The metal is 28 centimeters wide and 500 centimeters long. How much  $(x)$  should be turned up for each side in order for the gutter to hold the most water?

- 5 (a) State the Euler Theorem. If  $u = \ln \left( \frac{x^5 - y^5}{x + y} \right)$ , show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 4$  (05)

- (b) Approximate the change in the hypotenuse of a right angle triangle of legs 6 and 8 units. When the shorter leg is increased by 1/2 units and long leg is decreased by 1/4 units. (05)

- (c) Define Maxima and Minima of  $f(x, y) = 0$  and its working rules. (06) ABC  
 builders advertise for some housing project. If  $x, y$  represent the amount (in thousands of rupees) spent on newspaper and T.V advertisements respectively. The company's profit based on advertisement is given by

$$P(x, y) = -2x^2 - xy - y^2 + 8x + 9y + 10$$

How much amount should the company spend on each type of advertisement in order to maximize the profit and what will be the maximum profit?

- 6 Evaluate any THREE of the following integrals: (16)

$$(i) \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx \quad (ii) \int \frac{xdx}{(x+4)\sqrt{(x-1)}} \quad (iii) \int_0^1 \frac{1-x^2}{(1+x^2)^2} dx \quad (iv) \int \frac{xdx}{x^4 - x^2 - 1}$$

- 7 (a) The rate at which petroleum consumed in Pakistan was approximately  $c'(t) = 21t + 281$  million barrels per year from 1983 ( $t=0$ ) to 1987 ( $t=4$ ). Determine the total amount of petroleum consumed from 1983 to 1987. (05)

- (b) Determine the area enclosed between the two curves  $y = x + 5, y = \sqrt{x}$  from  $x = 0$  to  $x = 4$ . Show the area graphically. (06)

- (c) Evaluate  $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$  (05)

8. Do as directed. (Any Four) (16)

- (i) The out put  $P$  of battery is given by  $P = VI - RI^2$ , where  $V$  is voltage in volts,  $I$  is current in amperes and  $R$  is resistance in ohms. Find the maximum current in battery if  $V = 4$  volts and  $R = 8$  ohms.

(ii) Find the volume of parallelepiped whose adjacent sides are represented by the vectors  $\vec{a} = 3\hat{i} + 4\hat{j}$ ,  $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ , and  $\vec{c} = \hat{i} + 3\hat{j} + 5\hat{k}$ .

(iii) Find a unit vector normal to the surface  $\phi(x, y, z) = x^3y^2 - z^2yx^2 - 9$  at  $(1, 2, 1)$ .

(iv) Find the directional derivative of  $\phi(x, y, z) = 4x^2y^2z^2$  at  $(1, 2, 1)$  in the direction of  $2\hat{i} + \hat{j} + 2\hat{k}$ .

(v) For what value of constant “C”, the vector function

$\vec{f} = (3x + y)\hat{i} + (Cy + z)\hat{j} + 2z\hat{k}$  is Solenoidal.

(vi) A particle moves so that its displacement at time  $t$  is given by:

$\vec{x}(t) = 2\cos t\hat{i} + 2\sin t\hat{j} + t\hat{k}$ . Find the magnitude of the velocity and acceleration of the particle at  $t = 0$ .

-----THE END-----



MEHRAN UNIVERSITY OF ENGINEERING AND TECHNOLOGY, JAMSHORO.  
 DEPARTMENT OF BASIC SCIENCES AND RELATED STUDIES.  
 1<sup>ST</sup> TERM 1<sup>ST</sup> YEAR REGULAR EXAMINATION 2008  
 DISCIPLINES: ES-I      BATCH:08      MAX. MARKS: 80  
 TIME: 03 HOURS      DATED: 00-00-2008

**NOTE: ATTEMPT ANY FIVE QUESTIONS. MARKS ARE LISTED AGAINST EACH QUESTION**

**QUESTION** **MARKS**  
 Q#1(a) Evaluate any TWO of the following limits: **(06)**

(i)  $\lim_{x \rightarrow 2-0} \sqrt{\frac{4-x^2}{6-5x+x^2}}$       (ii)  $\lim_{x \rightarrow 0} \frac{e^{x^2}-1}{\cos x - 1}$       (iii)  $\lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^{\tan x}$

Q#1(b) The flow of current is given as under: **(05)**

$$\begin{aligned} I(t) &= t^2 + 1, & 0 < t < 1 \\ &= 2t, & 1 < t < 2 \\ &= t^2 - 1, & t > 2 \end{aligned}$$

where t is the time in seconds. Does the current flow continuous at t = 1, at t = 2. What will be the value of current at t = 5 seconds?

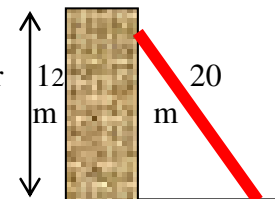
Q#1(c) A dynamite blasts and bellows a heavy rock. The rock reaches at a height of **(06)**  
 $h(t) = 12t^2 - 48t + 10$  feet, where t is the time in seconds. How high the rock goes?

Q#2(a) A manufacturer of telephones determines that the cost of producing x **(06)**  
 telephones is  $C(x) = 500 + 15x - 0.01x^2$  dollars and revenue from the sale of x telephones is  $R(x) = 75x - 0.02x^2$  dollars. Determine the marginal profit. What production level results in a marginal profit to zero?

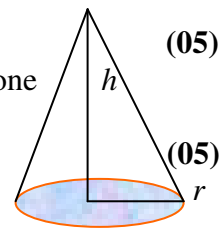
Q#2(b) Differentiate  $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$  w. r. t  $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$  **(05)**

Q#2(c) State the Leibniz's theorem. Use it to to prove that if  $y = (x + \sqrt{1+x^2})^m$  **(05)**  
 then  $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$

Q#3(a) A 20 meter ladder is being leaning against a wall. Suppose the ladder is **(06)**  
 sliding Down the wall in such a way that the bottom of the ladder is moving away from the wall at a rate of 3 m/s. At what rate is the top of the ladder sliding down the wall when the top of the ladder is 12m above the ground?



Q#3(b) Leaking sand forms a conical pile in which the height is always thrice the radius ( $h = 3r$ ). Consider the moment when the radius is 9 cm. Use differential to determine the approximate change in the volume when the radius changes by 1%, and volume of the cone is  $V = \pi r^2 h/3$ . **(05)**



Q#3(c) When a pill such as aspirin is swallowed; the concentration of the medicine in the blood-stream begins at zero and increases towards a maximum concentration. **(05)**

The concentration then declines until there is none of the medicine present. Suppose the concentration K of a particular medicine t hours after being swallowed is  $K = 0.03t/(1+t^2)$ . What will be highest concentration and after how many hours it occurs.

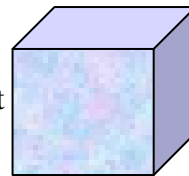
Q#4(a) State Rolle's theorem and give its geometrical interpretation. Verify it **(05)**

for the function  $f(x) = x(x-3)e^{2x}$  on  $[0, 3]$ .

Q#4(b) Define “Curvature” and “Radius of Curvature”. Find the radius of curvature (05)  
for the curve  $x^2 y = 4(x^2 - y^2)$  at  $(6, 3)$ .

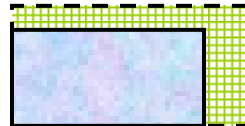
Q#4(c) State Euler’s theorem. Use it to show that if  $U = \log_e[(x^3 + y^3)/(x^2 + y^2)]$ , then (06)  
Then  $x U_x + y U_y = 1$ .

Q#5(a) A rectangular cardboard box with top closed is being made to contain a volume of  $27 \text{ ft}^3$ . Find the dimensions of the box that will minimize cost of material used to make the box.



(06)

Q#5(b) A rectangular plate expands in such a way that its length changes from 10” to 10.3” and its breadth changes from 8” to 8.2”. Find an approximate value for the change in its area? (05)



Q#5(c) If  $\mathbf{r} = t^3 \mathbf{i} + (2t^2 + 1) \mathbf{j} - t \mathbf{k}$ , evaluate  $\int_0^2 \bar{r} \times \frac{d^2 \bar{r}}{dt^2} dt$

(05)

Q#6(a) Define irrotational vector. Find the constants a, b, and c so that the vector  $\mathbf{V} = (x + 2y + az) \mathbf{i} + (bx - 3y - z) \mathbf{j} + (4x + cy + 2z) \mathbf{k}$  is irrotational. (06)

Q#6(b) Define a solenoid vector functions. Find the value of constant “C” so that the vector function  $\mathbf{f} = x^2 y^2 z \mathbf{i} + (x + C y^3) z^2 \mathbf{j} - xyz^3 \mathbf{k}$  is solenoid at  $(1, 2, 1)$ . (05)

Q#6(c) If  $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$  is a position vector, evaluate  $\nabla(\ln r)$ . (05)

Q#7(a) Evaluate any two of the following integrals: (06)

(i)  $\int \frac{1}{(x+1)\sqrt{x-1}} dx$  (ii)  $\int \frac{\cos x}{(\sin x + 1)(\sin x - 2)} dx$  (iii)  $\int x^2 \tan^{-1} x dx$

Q#7(b) Show that  $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \cos x}} dx = \frac{\pi}{4}$  (05)

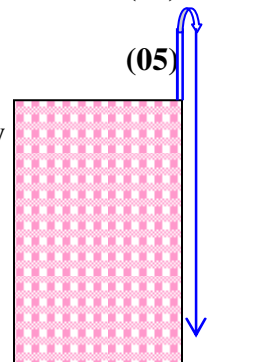
Q#7(c) Prove that  $\Gamma(1/2) = \sqrt{\pi}$ . Use the relation between “Beta” and “Gamma” functions  $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ , evaluate  $\beta(7/2, 9/2)$ . (05)

Q#8(a) Assume that a small tree will grow to maturity according to  $\frac{dh}{dt} = 2 + \frac{1}{2\sqrt{t}}$  (05)

where h is height in feet, t is time in years. How much does the tree grow between first and fourth year. How much does it grow between fourth and ninth year?

Q#8(b) Find the area between two curves  $y = x^2 + 2$  and  $y = 3x + 6$ . Show the area graphically? (06)

Q#8(c) A ball is shot vertically upward from the edge of a building with initial velocity 300 ft/s. the building is 700 feet tall. Acceleration due to gravity is  $-32 \text{ ft/s}^2$ . Determine the equations of the velocity and distance in terms of t. How far above the ground is the ball after 6 seconds and how fast is it going then? (05)



THE END

PRACTICAL TEST

**Q No.1 Fill IN THE BLANKS:**

**Marks 05**

- (i) If  $x$  is computed as 4.56 instead of 4.58, what is the percentage absolute error?
- (ii) Newton Raphson method is known as \_\_\_\_\_ method.
- (iii) The RF method is known as \_\_\_\_\_ method
- (iv) If  $Ax = \lambda x$ , then  $\lambda$  and  $x$  respectively called \_\_\_\_\_ & \_\_\_\_\_
- (v)  $\Delta^3 y_3 =$  \_\_\_\_\_
- (vi)  $\nabla^3 y_3 =$  \_\_\_\_\_
- (vii) Relation between  $\Delta$  and  $E$  is given by \_\_\_\_\_
- (viii) Relation between  $E$  and  $\nabla$  is given by \_\_\_\_\_
- (ix) If  $y_n = n^2$ , what is  $\Delta^2 y_n$
- (x) Picard's method is known as \_\_\_\_\_

**Q No. 2 TRUE OR FALSE:**

**Marks 05**

- (i) For Simpson's 3/8 rules number of sub-intervals must be an even.
- (ii) RF method is known as bracketing method.
- (iii) Fixed point iteration method  $x_{n+1} = g(x_n)$  converges if  $|g'(x_0)| \leq 1$ .
- (iv) N-R method converges fast if  $f'(x_0) \neq 0$ .
- (v) A Euler's method is analytical method for solving differential equation of 1<sup>st</sup> order.
- (vi) Euler Method for solving the 1<sup>st</sup> order differential equation is the most popular method.
- (vii) RK methods give better approximation than all other methods used for solving differential equations of order one.
- (viii) Lagrange Interpolation method is also applicable if the data values are equally spaced.
- (ix) Newton divided difference interpolation formula is better and easy to use than the Lagrange method.
- (x) Partial differential equation  $A U_{xx} + B U_{xy} + C U_{yy} = D$ , where  $A, B, C$  and  $D$  are some functions of  $x, y, u, U_x$ , and  $U_y$  is Parabolic differential equation if \_\_\_\_\_.

THE END

MEHRAN UNIVERSITY OF ENGINEERING AND TECHNOLOGY, JAMSHORO.  
DEPARTMENT OF BASIC SCIENCES AND RELATED STUDIES.  
1<sup>ST</sup> TERM 3<sup>RD</sup> YEAR REGULAR EXAMINATION 2008  
SUBJECT: NACA BATCH:06 CE MAX. MARKS: 10  
TIME: 1/2 HOURS DATED: 00-00-2008

PRACTICAL TEST

**Q No.1 Fill IN THE BLANKS:**

**Marks 05**

- (i) If x is computed as 7.56 instead of 7.59, what is the percentage absolute error?
- (ii) Bisection method is known as \_\_\_\_\_
- (iii)  $\Delta^3 y_5 =$  \_\_\_\_\_
- (iv)  $\nabla^3 y_6 =$  \_\_\_\_\_
- (v) Relation between  $\Delta$  and E is given by \_\_\_\_\_
- (vi) Relation between E and  $\nabla$  is given by \_\_\_\_\_
- (vii) If  $y_n = n^2$ , what is  $\Delta^2 y_n$
- (viii) Power method is used to compute \_\_\_\_\_ & \_\_\_\_\_ of a square matrix A.
- (ix) Newton Interpolation formulas are better than \_\_\_\_\_ and \_\_\_\_\_.
- (x) For trapezoidal rule the number of sub-intervals may be \_\_\_\_\_

**Q No. 2 TRUE OR FALSE:**

**Marks 05**

- (i) Simpson's 1/3 rule requires even value of n where n is number of sub-intervals.
- (ii) Newton-Raphson method is known as open method.
- (iii) Fixed point iteration method  $x_{n+1} = g(x_n)$  converges if  $|g'(x_0)| > 1$ .
- (iv) N-R method converges faster than RF method.
- (v) Euler method is analytical method for solving differential equation of 1<sup>st</sup> order.
- (vi) Euler Method for solving the 1<sup>st</sup> order differential equation is the most popular method.
- (vii) R-K methods give better approximation than Euler method.
- (viii)  $|A - \lambda I| = 0$  is said to be \_\_\_\_\_.
- (ix) The roots of characteristic equation are known as \_\_\_\_\_.
- (x) Lagrange Interpolation polynomial  $L_i(x)$  is given by:  
\_\_\_\_\_

THE END